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Short communication

CASTELO: Convex Approximation based Solution To Elliptic Localization with Outliers

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ABSTRACT

This short communication considers mitigating the negative effects of possibly unreliable path delay measurements acquired in non-line-of-sight (NLOS) environments on the positioning performance, a problem deserving further investigation within the expanding research area of elliptic localization. We present CASTELO, a *Convex Approximation based Solution To Elliptic Localization with Outliers*, to achieve such a goal. Our proposal corresponds to a mixed semidefinite (SD)/second-order cone (SOC) programming formulation derived from an error-mitigated nonlinear least squares (LS) location estimator, presenting itself as a remedy for the neglect of positivity of NLOS biases suffered by the majority of currently fashionable outlier-handling approaches. In terms of analytical discussions, we provide rationales supporting the incorporation of the SOC constraints, which serve to tighten the problem obtained after SD relaxation, and conduct a complexity analysis for the ultimate mixed SD/SOC programming formulation. Simulations are carried out to confirm the strong ability of CASTELO to attain reliable elliptic localization in the presence of NLOS outliers.

1. Introduction

Elliptic localization has lately become a central topic across the fields of distributed multiple-input multiple-output (MIMO) radar, sonar, and wireless sensor networks (WSNs) [1–3]. Such a positioning scheme makes use of spatially separated transmitters and receivers to find the location of a target, $\boldsymbol{x} \in \mathbb{R}^H$, in H-dimensional space ($H = 2$ or 3). In a cooperative (resp. noncooperative) manner, the signal emitted from each transmitter will be relayed (resp. reflected) by the target of interest and then picked up by the receivers. The corresponding indirect path delays, also known as (a.k.a.) the bistatic ranges (BRs), are acquired and utilized together with the known sensor positions in determining $\boldsymbol{x}$.

A procedure of least squares (LS) will normally be needed to make statistical sense out of the sensor-collected erroneous measurement data [2,3]. The key premise of the feasibility of LS techniques is that the disturbances should be zero-mean Gaussian distributed, namely, attributed to thermal fluctuations. However, the Gaussian noise assumption can be easily violated in real-world situations, where adverse environmental factors may come into play and, as a result, the sensor observations will be significantly biased [4]. Perhaps the most frequently encountered phenomenon in this context giving rise to outliers is the non-line-of-sight (NLOS) propagation of signals [5,6], which will introduce a positive distance bias into the affected range measurement and, eventually, greatly impairing the positioning accuracy.

Along the simplest line of thought, identify-and-discard (IAD) type approaches may be devised to pick out the corrupted samples and remove them from the data. For elliptic localization, this was accomplished in [7] based on the spectral graph theory, and in [8] with two different data selection procedures. Nonetheless, the simplicity of the IAD methodology comes at the costs of inevitable missed-detection and/or false-alarms [4]. Unlike [7,8], the authors of [9–11] employed the concept of robust statistics, to partly mitigate the negative impacts of outliers on the performance of the elliptic location estimator. The main idea is to replace the non-outlier-resistant ℓ_2 loss in the LS formulation with candidates that can be less influenced by the large fitting errors, e.g., the correntropy measure [9] and the ℓ_1 loss [10, 11]. A half-quadratic (HQ) algorithm was developed in [9] to tackle

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the complicated optimization problem established following the maximum correntropy criterion (MCC), whereas ℓ_1-norm minimization has been performed by means of the Lagrange programming neural network (LPNN) [10] and iterative message passing (MP) [11], respectively. Despite their superior outlier-resistance compared to the non-robust LS schemes, all these statistically robustified elliptic location estimators overlooked a crucial aspect: the NLOS propagation of signals always tends to positively bias the range-based measurements, and it has actually become commonplace for localization practitioners to take advantage of this positivity towards even better positioning performance [4,5,12–14].

This short communication puts forward CASTELO, a *Convex Approximation based Solution To Elliptic Localization with Outliers*, to tackle the open challenge of overlooking the positive biasedness properties in NLOS environments during the derivations of many existing error-mitigated elliptic location estimators. In simple terms, CASTELO employs mixed semidefinite (SD)/second-order cone (SOC) programming to deal with a nonconvex nonlinear LS problem established by introducing additional bias-representing estimation variables and integrating new designs for the relationship between them and the original NLOS biases. The remainder of this contribution is organized as follows. Section 2 describes the system model. The essence of CASTELO is subsequently unveiled in Section 3, in which we formulate elliptic localization in NLOS scenarios as an error-mitigated LS problem and then frame it within a mixed SD/SOC programming framework. The rationale for incorporating the SOC constraints and the complexity of CASTELO are also analyzed. Simulation results are included in Section 4. Finally, Section 5 concludes the paper.

2. System model

Elliptic localization refers to estimating the unknown position of a signal-reflecting/relaying target, $\boldsymbol{x}$, from the indirect path delay/BR measurements:

$$\hat{r}_{m,l} = \|\boldsymbol{x} - \boldsymbol{t}_m\|_2 + \|\boldsymbol{x} - \boldsymbol{s}_l\|_2 + p_{m,l}, m = 1, \dots, M, l = 1, \dots, L \tag{1}$$

obtained by employing multiple transmitters and receivers, which are known to be located at $\{\boldsymbol{t}_m\}$ and $\{\boldsymbol{s}_l\}$, respectively [3]. Here, $\|\cdot\|_2$ is the Euclidean norm and $\{p_{m,l}\}$ are the BR measurement errors.

As the emphasis of this work is on elliptic localization in NLOS situations, the measurement errors $\{p_{m,l}\}$ are further decomposed as

$$p_{m,l} = n_{m,l} + b_{m,l}, \tag{2}$$

where $n_{m,l}$ represents the lower-level Gaussian noise induced by thermal effects, and $b_{m,l}$ is a random variable that denotes the NLOS bias due to obstruction in the (m,l) transmitter–target–receiver path.

The system model described in (1) and (2) serves as the foundation for our work, and whether during the estimator derivations or in the simulations, we will consistently adhere to and stay within the scope defined by it.

In the literature, various statistical characterizations exist for $b_{m,l}$, such as those employing uniform, Gaussian, and exponential distributions [12]. Nevertheless, we will refrain from imposing any specific distribution constraints on $b_{m,l}$ during the derivation of our mixed SD/SOC programming based LS solution. It is, only when a particular distribution needs to be selected for the purpose of generating $b_{m,l}$ from a simulation standpoint, that we will opt for a specific choice (e.g., uniform, Gaussian, or exponential). In contrast, the estimator derivations will operate under the following three assumptions: (i) decomposability of $p_{m,l}$ into $n_{m,l}$ and $b_{m,l}$, (ii) Gaussianity of $n_{m,l}$, and (iii) bias-like property of $b_{m,l}$ that it manifests as a positive bias with a magnitude generally much greater than $|n_{m,l}|$ under NLOS conditions and simply reduces to zero under line-of-sight (LOS) conditions.

3. Mixed SD/SOC programming based LS solution for NLOS mitigation in elliptic localization

3.1. Formulation derivations

We start our derivations of the error-mitigated LS formulation by reshaping the BR model as

$$\hat{r}_{m,l} - d^t_m = d^s_l + b_{m,l} + n_{m,l},\ m = 1, \dots, M,\ l = 1, \dots, L, \tag{3}$$

where $d^t_m = \|\boldsymbol{x} - \boldsymbol{t}_m\|_2$ and $d^s_l = \|\boldsymbol{x} - \boldsymbol{s}_l\|_2$ are dummy variables introduced for the corresponding true target-sensor distances.

If we neglect second-order terms in $n_{m,l}$, squaring both sides of (3) will yield

$$\hat{r}^2_{m,l} - 2\hat{r}_{m,l} d^t_m + (d^t_m)^2 \approx (d^s_l)^2 + c_{m,l} + 2n_{m,l}(d^s_l + b_{m,l}),\quad m = 1, \dots, M,\ l = 1, \dots, L, \tag{4}$$

where

$$c_{m,l} = b^2_{m,l} + 2b_{m,l} d^s_l. \tag{5}$$

To simplify the resulting formulation, we treat $\{c_{m,l}\}$ instead of $\{b_{m,l}\}$ as the variables for biases associated with the indirect path delays, and disregard the equality constraints in (5) describing the relationship between $\{c_{m,l}\}$, $\{b_{m,l}\}$, and $\{d^s_l\}$. In so doing, our error-mitigated nonlinear LS problem can be built upon (4) as

$$\min_{\boldsymbol{x},\{c_{m,l}\}} \sum_{m=1}^{M} \sum_{l=1}^{L} w_{m,l} \left(\hat{r}^2_{m,l} - 2\hat{r}_{m,l} d^t_m + (d^t_m)^2 - (d^s_l)^2 - c_{m,l}\right)^2, \tag{6}$$

where a weighting factor $w_{m,l}$ is introduced to take into consideration the disparity in $n_{m,l}$ between different paths. In a maximum likelihood sense, it should be equal to the inverse of the variance of $2n_{m,l}(d^s_l + b_{m,l})$ [15]. Nonetheless, since $\{d^s_l\}$ are dependent on the unknown target location and even the *a priori* statistical knowledge of $\{n_{m,l}\}$ could be extravagant in practice, we somewhat compromise on such issues and follow [4] to set $w_{m,l} = 1 - \hat{r}_{m,l} / \sum_{m=1}^{M} \sum_{l=1}^{L} \hat{r}_{m,l}$, i.e., assigning more confidence to the nearby/LOS-prone links.

The nonlinear LS estimator (6) is nonconvex because of the Euclidean norm terms incorporated by the dummy variables $\{d^t_m\}$ and $\{d^s_l\}$. Furthermore, in regard to the newly-introduced variables $\{c_{m,l}\}$ that represent the transformed biases, the problem (6) is not well-posed since the relationship (5) is discarded. To tackle these challenges, we apply approximations to handle the nonconvexity and put forth additional designs for the free parameters $\{c_{m,l}\}$, resulting in the following convex programming problem:

$$\min_{\boldsymbol{x},\{c_{m,l}\},\{d^t_m\},\{g^t_m\},\{g^s_l\},\{u_{m,l}\},z} \sum_{m=1}^{M} \sum_{l=1}^{L} \Big[w_{m,l} \left(\hat{r}^2_{m,l} - 2\hat{r}_{m,l} d^t_m + g^t_m - g^s_l - c_{m,l}\right)^2 + \lambda_1 \left(c^2_{m,l} + (d^t_m)^2\right) + \lambda_2 u^2_{m,l} \Big],$$

$$\text{s.t. } c_{m,l} \geq 0,\ m = 1, \dots, M,\ l = 1, \dots, L, \tag{7a}$$

$$g^t_m = \begin{bmatrix} \boldsymbol{t}_m \\ -1 \end{bmatrix}^T \begin{bmatrix} \boldsymbol{I}_H & \boldsymbol{x} \\ \boldsymbol{x}^T & z \end{bmatrix} \begin{bmatrix} \boldsymbol{t}_m \\ -1 \end{bmatrix},\ m = 1, \dots, M, \tag{7b}$$

$$g^s_l = \begin{bmatrix} \boldsymbol{s}_l \\ -1 \end{bmatrix}^T \begin{bmatrix} \boldsymbol{I}_H & \boldsymbol{x} \\ \boldsymbol{x}^T & z \end{bmatrix} \begin{bmatrix} \boldsymbol{s}_l \\ -1 \end{bmatrix},\ l = 1, \dots, L, \tag{7c}$$

$$\begin{bmatrix} \boldsymbol{I}_H & \boldsymbol{x} \\ \boldsymbol{x}^T & z \end{bmatrix} \succeq \boldsymbol{0}_{(H+1)\times(H+1)}, \tag{7d}$$

$$\hat{r}^2_{m,l} - 2\hat{r}_{m,l} d^t_m + g^t_m + u_{m,l} \geq g^s_l,\ u_{m,l} \geq 0,\quad m = 1, \dots, M,\ l = 1, \dots, L, \tag{7e}$$

$$\|\boldsymbol{x} - \boldsymbol{t}_m\|_2 \leq d^t_m,\ m = 1, \dots, M, \tag{7f}$$

where $\boldsymbol{I}_i$ and $\boldsymbol{0}_{i\times j}$ denote the $i \times i$ identity matrix and the $i \times j$ zero matrix, respectively.

Now, let us elucidate the formulation (7), which we refer to as CASTELO, in more detail. In (7a), nonnegativity constraints are imposed upon the bias-representing variables $\{c_{m,l}\}$ by making use of the positivity of the decomposed NLOS bias term $b_{m,l}$. The SD cone constraints in (7b) and (7c) are derived from

$$g_m^t = \left(d_m^t\right)^2 = \|\mathbf{x} - \mathbf{t}_m\|_2^2 \tag{8}$$

and

$$g_l^s = \left(d_l^s\right)^2 = \|\mathbf{x} - \mathbf{s}_l\|_2^2, \tag{9}$$

respectively, where $\{g_m^t\}$ and $\{g_l^s\}$ are the second-order versions of the corresponding distance-representing variables. SD relaxation takes place in (7d), achieved by discarding the rank-1 requirement for the nonconvex equality constraint $z = \mathbf{x}^T\mathbf{x}$ characterizing the relationship between the location-representing vector variable $\mathbf{x}$ and a new variable z introduced to denote its squared Euclidean distance. The inequality constraints in (7e) are general consensus nonnegativity constraints based on the one-sided biasing property of the NLOS errors just like (7a), except that they are now imposed in the quadratic form:

$$(\hat{r}_{m,l} - d_m^t)^2 \geq (d_l^s)^2 = g_l^s,\ \ m = 1,\dots,M, l = 1,\dots,L. \tag{10}$$

$\{u_{m,l}\}$ are variables added to these quadratic nonnegativity constraints for slightly loosening them in possibly infeasible cases [14]. The hyperparameters λ_1 and λ_2 are both penalization factors. Regarding λ_1, it is required when the problem is ill-posed, and it helps constrain the newly-introduced bias-representing variables $\{c_{m,l}\}$ to a reasonable size [16]. On the other hand, λ_2 safeguards against overly loose nonnegativity constraints in (7e). We will delve into the impact of these hyperparameters, numerically, in Section 4. Additionally, the SOC constraints in (7f) are deduced from the transmitter–target distance relationship: $d_m^t = \|\mathbf{x} - \mathbf{t}_m\|_2$, $m = 1,\dots,M$. Their inclusion is motivated by the potential of such SOC constraints to tighten the problem obtained after a straightforward SD relaxation [17], as outlined in the proposition below.

Proposition 1. *The constraints in (7f) are tighter than those defined by (7b) and (7d).*

Proof. Employing the Schur complement condition [16], we have the following equivalence for (7d):

$$\begin{bmatrix} \mathbf{I}_H & \mathbf{x} \\ \mathbf{x}^T & z \end{bmatrix} \succeq \mathbf{0}_{(H+1)\times(H+1)} \Longleftrightarrow \mathbf{x}^T\mathbf{x} \leq z. \tag{11}$$

It then becomes evident that, through (7b) and (7d), the following are being imposed:

$$g_m^t = \|\mathbf{t}_m\|_2^2 - 2\mathbf{t}_m^T\mathbf{x} + z \geq \|\mathbf{x} - \mathbf{t}_m\|_2^2,\ \ m = 1,\dots,M. \tag{12}$$

The inequality constraints in (12), when considered along with[2]

$$(d_m^t)^2 \leq g_m^t,\ \ m = 1,\dots,M, \tag{13}$$

describe how the SD relaxation is typically applied in such a context [18,19]. Clearly, (12) and (13) define weaker conditions compared to (7f), and this justifies our choice to impose (7f) instead of (13) in our CASTELO formulation. □

To enhance clarity of presentation, we bring in Table 1 to explicitly outline the definition of the variables involved with (7).

[2] The constraints in (13) can be derived from (8) [18,19].

Table 1
Definition of variables in (7).

Variable	Definition
$\mathbf{x}$	Target location
$\mathbf{t}_m$	Location of the mth transmitter
$\mathbf{s}_l$	Location of the lth receiver
$c_{m,l}$	Transformed bias associated with the (m,l) transmitter–target–receiver path
d_m^t	Euclidean distance between the mth transmitter and the target
g_m^t	Squared Euclidean distance between the mth transmitter and the target
g_l^s	Squared Euclidean distance between the target and the lth receiver
$u_{m,l}$	Small parameter added to the quadratic nonnegativity constraint associated with the (m,l) transmitter–target–receiver path for slightly loosening it in possibly infeasible cases
z	Squared Euclidean distance for $\mathbf{x}$
$w_{m,l}$	Weighting factor introduced to take into consideration the disparity in Gaussian noise between different paths and assign more confidence to the nearby/LOS links [4]
$\hat{r}_{m,l}$	BR measurement associated with the (m,l) transmitter–target–receiver path
λ_1	Penalization factor required when (7) is ill-posed, which helps constrain $\{c_{m,l}\}$ to a reasonable size [16]
λ_2	Penalization factor that safeguards against overly loose nonnegativity constraints in (7e)

3.2. Complexity analysis

The worst-case complexity for implementing mixed SD/SOC programming via a generic interior-point algorithm is on the order of [6]

$$\sqrt{\sum_{j=1}^{N_{\mathrm{SD}}} D_{\mathrm{SD},j} + 2N_{\mathrm{SOC}}}\ \Bigg(N_{\mathrm{Var}} \sum_{j=1}^{N_{\mathrm{SOC}}} D_{\mathrm{SOC},j}^2 + N_{\mathrm{Var}}^2 \sum_{j=1}^{N_{\mathrm{SD}}} D_{\mathrm{SD},j}^2 + N_{\mathrm{Var}} \sum_{j=1}^{N_{\mathrm{SD}}} D_{\mathrm{SD},j}^3 + N_{\mathrm{Var}}^3 \Bigg) \log\left(\frac{1}{\nu}\right), \tag{14}$$

where N_{Var}, N_{SOC}, N_{SD}, $D_{\mathrm{SOC},j}$, $D_{\mathrm{SD},j}$, and ν represent the number of the optimization variables, the number of the SOC constraints, the number of the SD cone constraints, the dimension of the jth SOC constraint, the dimension of the jth SD cone constraint, and the precision, respectively. As there are in total $N_{\mathrm{Var}} = H + 2M + L + 2ML + 1$ optimization variables, $3ML + M + L$ SD cone constraints of size 1 (corresponding to the constraints in (7a), (7b), (7c), and (7e)), 1 SD cone constraint of size $H + 1$ (corresponding to the constraint in (7d)), and M SOC cone constraints of size $H + 1$ (corresponding to the constraints in (7f)), the worst-case computational complexity of solving (7) with interior-point methods is $\mathcal{O}((ML)^{3.5})$. The LPNN (termed $\ell_1 - \mathtt{LPNN}$) and MP (termed $\mathtt{MP}$) algorithms adopted in the two representative existing works [10,11] within the field of statistically robustified elliptic localization, in contrast, result in $\mathcal{O}(N_{\mathrm{LPNN}}(M + L))$ and $\mathcal{O}(N_{\mathrm{MP}}ML)$ complexity, respectively, where N_{LPNN} is the number of steps required for discretely implementing the LPNN and N_{MP} the number of MP iterations.

To summarize, the computational cost of implementing CASTELO is higher than those of several existing robust statistical approaches. This difference in complexity is attributed to the fact that among the approaches we consider, our mixed SD/SOC programming based LS solution is the sole convex programming based technique implemented using interior-point methods. On the contrary, both benchmarking schemes are lightweight nonlinear location estimators directly implemented through advanced iterative optimization solvers. Such a

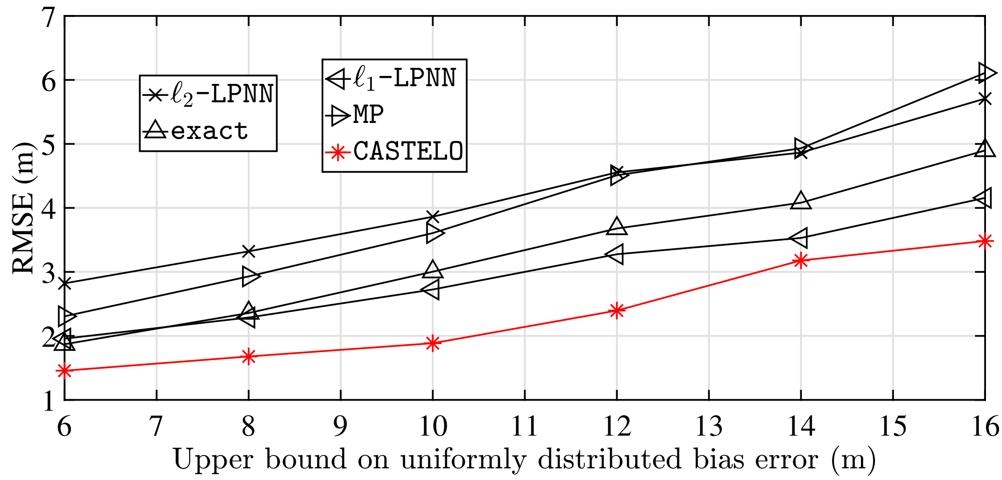

Fig. 1. RMSE versus upper bound on uniformly distributed bias error.

disparity between convex programming based methods and direct nonlinear methods is not uncommon and has been widely observed in related research[3] [4,13,20].

4. Numerical results

This section carries out numerical studies to evaluate the performance of the presented mixed SD/SOC programming based nonlinear LS solution for NLOS mitigation in elliptic localization (termed CASTELO, code available at https://github.com/w-x-xiong/PEP-SDP1). The multistatic system being simulated consists of $M = 3$ transmitters, $L = 4$ receivers, and a target to be located, all of which are deployed in two-dimensional (2-D) space (to accommodate the hard-coded 2-D configuration used in some of the existing elliptic location estimators). Specifically, their positions are set as $\boldsymbol{x} = [100, 100]^T$ m, $\boldsymbol{t}_1 = [-200, -300]^T$ m, $\boldsymbol{t}_2 = [-200, 300]^T$ m, $\boldsymbol{t}_3 = [200, 300]^T$ m, $\boldsymbol{s}_1 = [-450, -450]^T$ m, $\boldsymbol{s}_2 = [450, 450]^T$ m, $\boldsymbol{s}_3 = [0, 600]^T$ m, and $\boldsymbol{s}_4 = [600, 0]^T$ m. We conduct comparisons among CASTELO, ℓ_1-LPNN, MP, and several typical elliptic localization approaches without special designs for handling outliers, including the LPNN in [2] (termed ℓ_2-LPNN) and the exact solution in [3] (termed exact). The localization accuracy is assessed via the root-mean-square error (RMSE) computed based on 100 Monte Carlo (MC) runs: $\sqrt{\frac{1}{100}\sum_{i=1}^{100}\|\tilde{\boldsymbol{x}}^{\{i\}} - \boldsymbol{x}^{\{i\}}\|_2^2}$, where $\tilde{\boldsymbol{x}}$ denotes the location estimate and $(\cdot)^{\{i\}}$ is the index of the MC samples. Simulations were performed on a laptop with a 2.8 GHz CPU and 8 GB of RAM. The convex optimization problem was solved using the SeDuMi solver embedded within the MATLAB CVX package [21]. Unless their impact on the performance of CASTELO is being evaluated, the two hyperparameters λ_1 and λ_2 are set to 0.01 and 1, respectively, the values of which are empirically observed to result in decent estimation performance. With regard to the strategy for generating the measurement noise in the simulations, we consider the lower-level disturbances $\{n_{m,l}\}$ to follow independent and identically distributed (i.i.d.) zero-mean Gaussian distributions with a constant standard deviation of 1 m, viz., $n_{m,l} \sim \mathcal{N}(0, 1)$, $\forall(m, l)$. On the other hand, we will explore various distributions to characterize the NLOS biases $\{b_{m,l}\}$ in the following three separate subsections, as previously discussed in Section 2.

4.1. Localization performance with *uniform* $\{b_{m,l}\}$

We start by borrowing ideas from [4,6] to model $\{b_{m,l}\}$ as i.i.d. uniform distributions. Typically, a lower bound and an upper bound should be provided to specify an interval from which random numbers following a continuous uniform distribution are generated. We set such a lower bound to zero and investigate the RMSE as a function of the upper bound. The results are demonstrated in Fig. 1. Observably, CASTELO has the best performance, outperforming not only the non-robust ℓ_2 schemes ℓ_2-LPNN and exact but also the off-the-shelf robust statistical approaches MP and ℓ_1-LPNN.

4.2. Localization performance with *Gaussian* $\{b_{m,l}\}$

Next, we adopt another widely used strategy [22] to model $\{b_{m,l}\}$ as i.i.d. non-zero-mean Gaussian distributions instead. As the BR measurement errors $\{p_{m,l}\}$ now manifest themselves as Gaussian mixtures with two components, we additionally introduce a mixture proportion parameter $\beta \in (0, 1)$ into the model, as per conventions commonly followed in the relevant literature [22]. This leads to the following specific decomposition of $e_{m,l}$:

$$e_{m,l} = \beta\mathcal{N}(0, 1) + (1 - \beta)\mathcal{N}(\mu_{\text{NLOS}}, \sigma_{\text{NLOS}}^2), \ \forall(m, l), \tag{15}$$

which will still adhere to the three assumptions we made during the estimator derivations if the mean of the NLOS-representing Gaussian component μ_{NLOS} and the standard deviation σ_{NLOS} are properly configured. With μ_{NLOS} and σ_{NLOS} fixed at 20 m and 1 m, respectively, Fig. 2 plots the RMSE versus the mixture proportion of the first Gaussian mixture component (namely, β). We observe that the performance of all five estimators improves as the first GM component gradually prevails (viz., as the environment changes from severe NLOS to mild NLOS). In particular, CASTELO delivers the smallest RMSE for the whole range of β being examined. The performance superiority of CASTELO is also evident in Fig. 3 demonstrating how the RMSE changes with $\mu_{\text{NLOS}} \in [10, 20]$ m, under the conditions $\beta = 0.5$ and $\sigma_{\text{NLOS}} = 1$ m.

4.3. Localization performance with *exponential* $\{b_{m,l}\}$

In this subsection, we use exponential distributions, one more frequently employed NLOS-modeling scheme [12], to characterize $\{b_{m,l}\}$. Assuming that $\{b_{m,l}\}$ are i.i.d. exponential processes, we vary their mean from 2 m to 10 m and evaluate the performance of different location estimators at these various levels of NLOS contamination. The corresponding RMSE plot is displayed in Fig. 4. Clearly, we can observe that CASTELO, once more, yields the lowest RMSE values among the five approaches.

[3] One perspective is to consider it as the trade-off for the benefit of convex programming based methods, which consistently deliver a global solution due to the convex nature of the problems they address, i.e., a trait not shared by their direct nonlinear estimator counterparts.

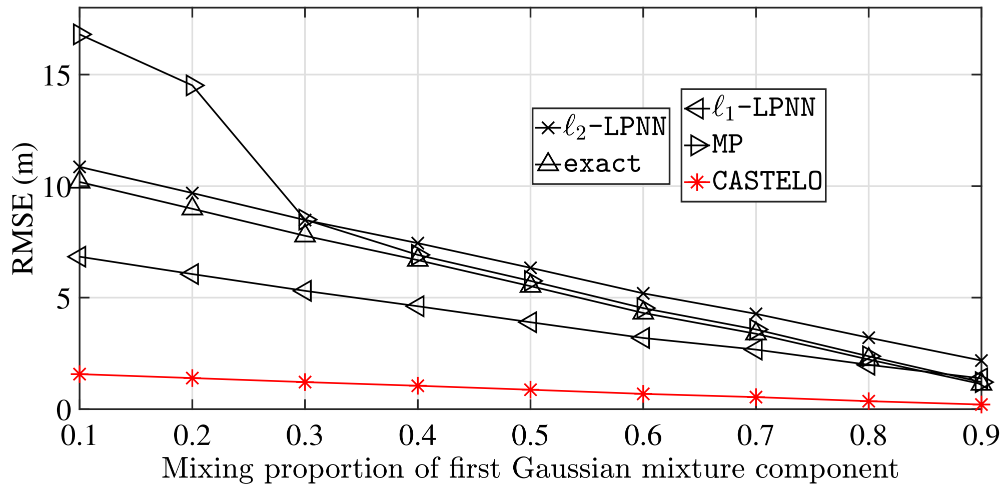

Fig. 2. RMSE versus mixing proportion of first Gaussian mixture component.

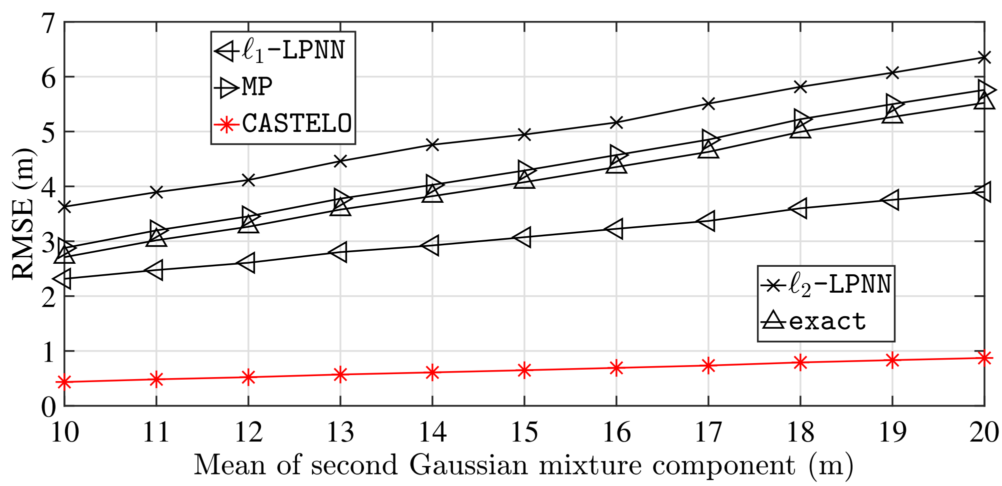

Fig. 3. RMSE versus mean of second Gaussian mixture component.

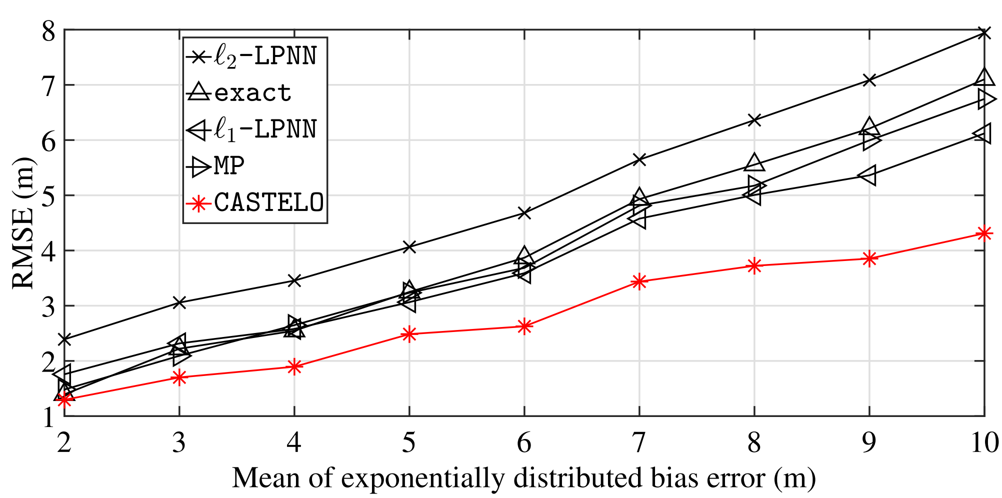

Fig. 4. RMSE versus mean of exponentially distributed bias error.

4.4. Computational complexity

We have recorded the average run-times taken by ℓ_2-LPNN, `exact`, ℓ_1-LPNN, MP, and CASTELO in the last three experimental subsections, which are 0.0034 s, 0.0238 s, 0.0037 s, 0.0003 s, and 0.6556 s, respectively. While CASTELO exhibits a longer run-time compared to the existing direct nonlinear methods, its practicality for location-based services is not entirely compromised (namely, a duration of around 0.6 s should still be deemed acceptable for location-based services of common purposes).

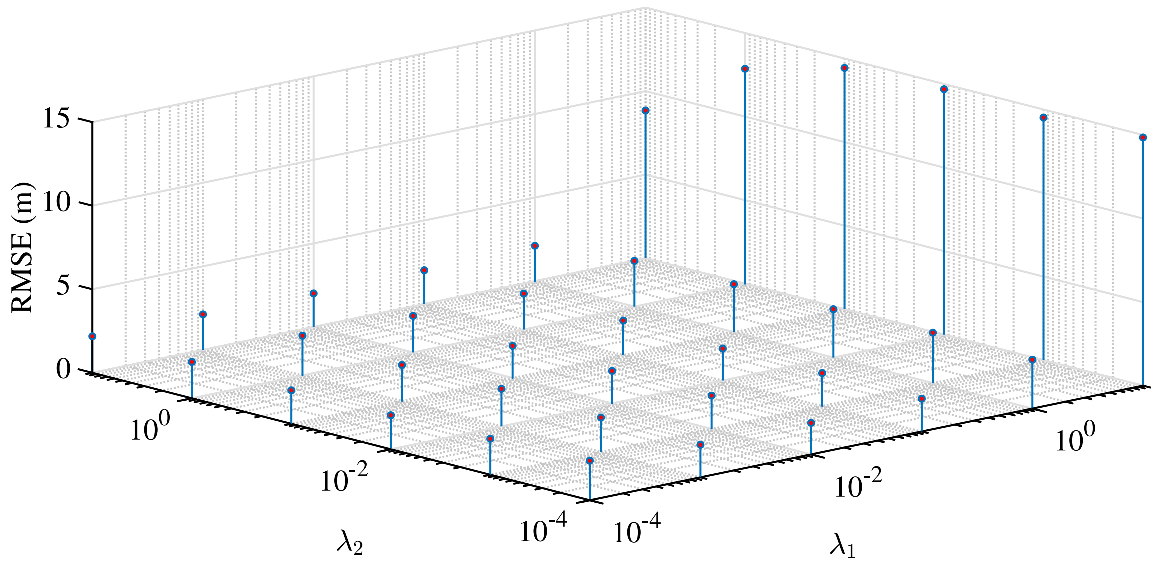

Fig. 5. CASTELO's RMSE variation with penalization factors λ_1 and λ_2.

Indeed, the computational complexity of our mixed SD/SOC programming based LS solution, owing to its convex programming nature, is not optimally low. In what follows, we offer several potential avenues towards reducing the complexity. First, in our numerical tests, the convex optimization problem was handled using the general problem solver `SeDuMi` integrated in the MATLAB `CVX` toolbox. While invoking `SeDuMi` incurs extra processing time due to the conversion of the input convex programming problem into the standard form, researchers have observed that this processing accounts for approximately half of the total CPU time [23]. If a convex programming solver specifically tailored for the problem is employed instead of the universal solver `SeDuMi`, the CPU time can be further decreased. Moreover, transitioning to the C/C++ programming language, as opposed to MATLAB which we consider here only for computer simulations, may also substantially enhance the computational speed in real-world operations.

4.5. Impact of penalization factors

An additional aspect worthy of investigation is the selection of the two penalization factors in the formulation (7) and, particularly, how it influences `CASTELO`'s estimation performance. To evaluate this, in Fig. 5, we utilize the experimental setup of Section 4.1 and present a three-dimensional stem graph depicting the RMSE variation for `CASTELO` with respect to $\lambda_1, \lambda_2 \in \{10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, 10^{0}, 10^{1}\}$. The upper bound on the i.i.d. uniformly distributed NLOS bias errors $\{b_{m,l}\}$ is fixed at 10 m. We observe that most (λ_1, λ_2) parameter settings, including the one we adopted in the previous experimental subsections (i.e., $\lambda_1 = 0.01$ and $\lambda_2 = 1$), demonstrate reasonably low RMSE values on a comparable scale. Nevertheless, configurations with an excessively large value for λ_1 lead to a noticeable deterioration in the estimator's performance.

5. Conclusion

This short communication identified an unresolved issue within the problem domain of elliptic localization in NLOS environments: the positivity of the NLOS bias errors was ignored in many of the current approaches for managing outliers in the BR measurements. In order to remedy such deficiencies and incompleteness, we put forward CASTELO, a convex approximation based solution to elliptic localization with outliers. CASTELO involves applying the technique of mixed SD/SOC programming to tackle an error-mitigated nonlinear LS estimation problem, which is established by introducing alternatives for the bias-representing variables and redefining the relationship between them and the original NLOS biases. Tightness and complexity analyses justified our incorporation of the SOC constraints and confirmed that CASTELO is in general computationally affordable, respectively. Numerical simulations were conducted to substantiate the effectiveness of CASTELO for NLOS-resistant elliptic localization. Prospective directions for future research include (i) carrying out real-world elliptic localization experiments and (ii) designing advanced outlier-handling schemes to better suit the actual elliptic localization conditions.

CRediT authorship contribution statement

Wenxin Xiong: Data curation, Formal analysis, Methodology, Software, Validation, Writing – original draft, Writing – review & editing. **Zhang-Lei Shi:** Investigation. **Hing Cheung So:** Supervision. **Junli Liang:** Conceptualization. **Zhi Wang:** Resources.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

We have shared the link to our code in the section of Numerical results.

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